Some of the formulas that we still use today were first devised and recorded in the 3rd century BCE, by the Greek mathematician Euclid of Alexandria in his 13 volume treatise *Elements* which served as the main textbook for teaching mathematics (especially geometry) from the time of its publication until the late 19th or early 20th century.

Operators of wastewater treatment plants need to be familiar with the formulas for calculating areas, perimeters and volumes of a variety of geometric shapes. The shapes described below can be found in treatment process tanks and basins, in clarifiers, lagoons, trenches, storage hoppers and a variety of other locations and applications.

As the picture above shows, operators of wastewater treatment plants may be called upon to calculate the area of rectangles (aeration basins, primary clarifiers) and circles (clarifiers); the volume of polyhedrons (aeration basins, primary clarifiers, etc.), cylinders (clarifiers, digestors) and occasionally, a sphere (gas holder) or a storage hopper with a conical bottom and cylindrical barrel. Linear measurements such as the amount of perimeter fencing required or the circumference of circular process tank must also be calculated from time to time.

The tools to carry out these calculations are presented in the remainder of this section.

Pi (π)

π (sometimes written **pi**) is a mathematical constant which equals the ratio of a circle's circumference to its diameter.

$$\pi = \frac{\text{circumference}}{\text{diameter}} \approx 3.14$$

Many formulas in mathematics, science, and engineering involve π, which makes it one of the most important mathematical constants.

Pi is an irrational number, which means that its value cannot be expressed exactly as a fraction having integers in both the numerator and denominator (for example, 22/7). Consequently, its decimal representation never ends and never repeats. Reports on the latest, most-precise calculation of π are common. The record as of July 2016, stands at 13 trillion decimal digits.

The value used for π in all calculations in this book and on the EOCP exams is 3.14

The constant 0.785

The number 0.785 often appears in formulas requiring the calculation of the area of a circle.

The equations: Area = $0.785(D)^2$ and Area = πr^2 will give the same answer. Why?

Proof:

If $\pi = 3.14$ and the radius of a circle is equal to one half the diameter i.e. $r = D/2$

$$\text{Then} \quad \text{Area} = \pi r^2 = \pi \left(\frac{D}{2}\right)^2 = \pi \frac{D^2}{4} = \frac{\pi D^2}{4} = \frac{3.14 D^2}{4} = 0.785 D^2$$

Because $3.14 / 4 = 0.785$

Both formulas are correct but to avoid confusion operators should chose to use one or the other in all of their calculations. In this manual, the formula $A = \pi r^2$ will be used throughout.

Perimeter and Circumference

A **perimeter** is a path that surrounds a two-dimensional shape. The word comes from the Greek *peri* (around) and *meter* (measure). The term may be used either for the path or its length—it can be thought of as the length of the outline of a shape

In the wastewater industry this term is usually applied to shapes which are square or rectangular. A rectangle is any four sided shape having at least 1 right angle and a length which is longer than its width. A square is any four sided shape having at least 1 right angle and all four sides equal in length.

A practical application may be the calculation of the linear metres of fencing required to enclose a space.

The formula for calculating the perimeter of a rectangle is:

$$\text{Perimeter} = 2 \times (\text{length} + \text{width})$$

It is written as:

$$P = 2 \times (L + W) \text{ or } P = 2(L + W) \text{ or } P = 2L + 2W$$

How many metres of fencing will be required to enclose a building lot that is 18 m wide by 45 m long?

Known: Length = 45 m, Width = 18 m

Insert known values and solve:

$$\mathrm{P} = 2 \times (\mathrm{L} + \mathrm{W}) = 2 \times (45\ \mathrm{m} + 18\ \mathrm{m}) = 2 \times (63\ \mathrm{m}) = 126\ \mathrm{metres}$$

The term **circumference** is used to refer to the distance around the outside of a circular or elliptical shape (its perimeter).

Calculation of the circumference of a circle requires the operator to know either its diameter (the distance across a circle at its widest point) or its radius (the distance from the center of a circle to its circumference or one half the diameter) and the value of the constant pi (3.14).

The formula for calculating the circumference of a circle is:

$$\mathrm{circumference} = \mathrm{pi}\ \times \mathrm{diameter}\ \text{ or }\ \mathrm{pi} \times 2 \times \mathrm{radius}$$

It is written as:

$$\mathrm{C} = \pi\mathrm{d}\ \text{ or } \mathrm{C} = 2\pi\mathrm{r}$$

There is no simple formula with high accuracy for calculating the circumference of an ellipse. There are simple formulas but they are not exact, and there are exact formulas but they are not simple. Thankfully, there are not many elliptical aeration basins being constructed. The most accurate of the simple formulae for the circumference of an ellipse is:

$$\mathrm{circumference} = \pi \times [3(\mathrm{a} + \mathrm{b}) - \sqrt{(3\mathrm{a} + \mathrm{b})(\mathrm{a} + 3\mathrm{b})}\ ]$$

Where “a” and “b” are the major and minor axes of the ellipse and “a” is not more than three time the length of “b”. Even then, the formula is only accurate to ±5%.

What is the circumference of a secondary clarifier with a diameter of 45 metres?

Known: Diameter = 45 metres, pi (π) = 3.14

Insert known values and solve:

$$C = \pi d = 3.14 \times 45 \text{ m} = 141.3 \text{ metres}$$

What is the circumference of a gravity thickener with a radius of 9 metres?

Known: radius = 9 metres, pi (π) = 3.14

Insert known values and solve:

$$C = 2\pi r = 2 \times 3.14 \times 9 \text{ m} = 56.52 \text{ metres}$$

Area

The area of a geometrical shape such as a circle, square, rectangle or triangle is the space contained within the boundary of the shape (i.e. its perimeter). Two dimensions are required to calculate the area of a shape and that area is reported as "units" squared. In the metric system the units that are most commonly used are the square metre (m^2) and the square centimetre (cm^2). Large shapes such as land surveys and wastewater lagoons are often reported in units of hectares (10,000 m^2).

Area of a Square or Rectangle

The area of a square or rectangle is equal to the product of one long side multiplied by one short side or in the case of a square by one side multiplied by another.

The formula for the area of a square or rectangle is:

$$\text{Area} = \text{Length} \times \text{Width}$$

It is written as:

$$A = L \times W \text{ or } A = LW \text{ or } A = (L)(W)$$

What is the surface area of a clarifier that is 8 metres wide and 50 metres long?

Known: Width = 8 metres, Length = 50 metres

Insert known values and solve;

$$\text{Area} = 8 \text{ m} \times 50 \text{ m} = 400 \text{ m}^2$$

Area of a Triangle

The area of a triangle is equal to its base (any side of the triangle) multiplied by its height (perpendicular to, or at 90° to the base), divided by two (often written as multiplication by ½).

The formula is

$$\text{Area} = \frac{\text{Base} \times \text{Height}}{2}$$

It is written as:

$$A = \frac{(B) \times (H)}{2} \quad \text{or} \quad \frac{1}{2} B \times H$$

A compost pile is 7 metres wide and 3 metres high. What is its cross-sectional area?

Known: Width (base) = 7 metres, Height = 3 metres

Insert known values and solve:

$$\text{Area} = \frac{7 \text{ m} \times 3\text{m}}{2} = 10.5 \text{ m}^2$$

Area of a Trapezoid

Calculating the area of a trapezoid falls somewhere between calculating the area of a square and calculating the area of a triangle. Trapezoidal shapes found in the industry include trenches dug for the installation of pipelines and stock piles of materials such as wood chips, compost or soil.

The area of trapezoid is equal to the sum of its two sides divided by 2 times its height. The formula is:

$$\text{Area} = \frac{\text{side 1} + \text{side 2}}{2} \times \text{height}$$

A pile of compost has a base 5 meters wide, a top 2.5 metres wide and a height of 2 meters. Calculate the cross-sectional area of the pile.

Known: side 1 = 5 m, side 2 =2.5 m, height = 2 m

Insert known values and solve

$$\text{Area} = \frac{\text{side 1} + \text{side 2}}{2} \times \text{height} = \frac{5 \text{ m} + 2.5 \text{ m}}{2} \times 2 \text{ m} = 7.5\text{m}^2$$

Area of a Circle

The area of a circle is equal to its radius (the distance from the center to any outside point of the circle) squared, multiplied by the constant π.

The formula is:

$$\text{Area} = \pi \times (\text{radius})^2 \ \text{ or } \ \text{Area} = \pi \times \text{radius} \times \text{radius}$$

It is written as:

$$A = \pi r^2$$

Calculate the surface area of a secondary clarifier which has a diameter of 45 metres.

Known: π = 3.14, Radius = ½ of the diameter = 22.5 metres

Insert known values and solve:

$$\text{Area} = \pi(r)^2 = 3.14 \times (22.5 \text{ m})^2 = 3.14 \times 22.5 \text{ m} \times 22.5 \text{ m} = 1{,}589.6 \text{ m}^2$$

Circular shapes found in the industry include clarifiers, thickeners, wet wells, meter vaults and pipes.

Area of a Cylinder

Calculating the area of a cylinder is a two-step operation. First the operator must calculate the circumference of the cylinder (i.e. the distance around the outside) and multiply that value by the height, depth or length of the cylinder as the case may be.

The practical application of this calculation is to determine the surface of area of a pipe, storage tank or reservoir in order to determine the quantity of paint or some other type of coating to be applied.

The equation is:

$$\text{Area} = \text{Circumference} \times \text{Height}$$

It is written as:

$$\text{Area} = \text{C} \times \text{H} \;\text{ or }\; \text{Area} = \pi \times \text{D} \times \text{H}$$

If the total area of a cylinder is to be calculated, as in calculating the surface area of a fuel tank then the two ends of the cylinder must also be accounted for and the formula becomes

$$\text{Total surface area} = \pi \times \text{D} \times \text{H} + 2 \times \pi \text{r}^2$$

A 600 mm diameter force main that is 1.2 km long needs to be coated with an epoxy paint. Calculate the number of square meters of pipe that require coating.

Step 1 – Convert to common units: 600 mm = 0.6 metres 1.2 km = 1,200 metres

Step 2 – Insert known values and solve

$$\text{Area} = \pi \times \text{D} \times \text{H} = 3.14 \times 0.6\ \text{m} \times 1{,}200\ \text{m} = 2{,}260.8\ \text{m}^2$$

Area of a Sphere

This formula is provided in the EOCP handout with the notation that it might be used to calculate the surface area of an air bubble. It could also be used to calculate the surface area of a gas holder associated with an anaerobic digestor.

The equation is:

$$\text{Area} = 4 \times \pi \times (\text{radius})^2$$

It is written:

$$\text{Area} = 4\pi\text{r}^2 \text{ or } \pi\text{d}^2 \text{ (where d} = \text{diameter)}$$

Area of a Cone

This formula is provided in the ABC Canadian handout but not the EOCP handout. Its practical application would be to calculate the surface area of a conical section of a hopper or the floor of a clarifier, trickling filter or anaerobic digestor in order to determine the amount of a coating needed.

The formula is:

$$\text{Area} = \pi \times \text{radius} \times \sqrt{(\text{radius})^2 + (\text{height})^2}$$

It is written:

$$\text{Area} = \pi \times \text{r} \times \sqrt{\text{r}^2 + \text{h}^2}$$

A gravity thickener 10 m in diameter has a cone shaped floor. The cone is 1.5 m deep. A skim coat of concrete is to be applied to the floor. Calculate the number of square metres to be covered.

Known: Radius = ½ of diameter = 5 metres, height = 1.5 metres

Insert known values and solve

$$\text{Area} = \pi \times \text{r} \times \sqrt{\text{r}^2 + \text{h}^2}$$

$$\text{Area} = 3.14 \times 5\text{m} \times \sqrt{(5\text{m})^2 + (1.5\text{m})^2}$$

$$\text{Area} = 3.14 \times 5\text{m} \times \sqrt{27.25\ \text{m}^2} = 82\ \text{m}^2$$

NOTE: *it is generally accepted that the math questions on a certification exam can be solved with a basic four function calculator, therefore, it is unlikely that any questions requiring the calculation of a square root will appear on the exam.*

Area of an Irregular Shape

Occasionally it is necessary to calculate the area of an irregular shape such as a sewage lagoon. One ways to do this is to break the shape into a number of shapes for which we have formulas (such as squares, rectangles or triangles). The area of each shape can be calculated, then added together to equal the area of the entire shape.

Volume

A measure of the three dimensional space enclosed by a shape. As volume is a three-dimensional measurement, the units used to describe it need to have three dimensions as well. These units are reported as "units" cubed or cubic "units". In the metric system volume is often expressed as cubic metres (m^3), cubic centimetres (cm^3) and liters (1,000 cm^3). Large volumes are also reported as Megaliters (1 ML = 1,000,000 L =1,000 m^3).

In the water and wastewater industry operators often need to calculate the volume of a basin (rectangular), clarifier, digestor or reservoir (cylinder), compost pile or stockpile (triangular) or a storage hopper (conical) or of a structure that is a combination of shapes (e.g. a digestor with a cylindrical body and a conical floor)

Volume of a Rectangular Tank

The volume of a box or cube is equal to its length, multiplied by its width, multiplied by its height, (depth or thickness). In the case of a cube, all three lengths are the same.

The formula is

$$\text{Volume} = \text{length} \times \text{width} \times \text{height}$$

It is written:

$$\text{V} = \text{LWH} \ \text{ or V} = (\text{L})(\text{W})(\text{H}) \ \text{ or } \ \text{V} = \text{L} \times \text{W} \times \text{H}$$

Sometimes the word "depth" and the letter "D" is substituted for height

Calculate the volume of an aeration basin 50 metres long by 6 metres wide by 4.5 metres deep.

Known: Length = 50 m, Width = 6 m, Depth = 4.5 m

Insert known values and solve:

$$V = LWD = 50 \text{ m} \times 6 \text{ m} \times 4.5 \text{ m} = 1{,}350 \text{ m}^3$$

Volume of a Prism

The mathematical name for a three-dimensional shape that is triangular in cross-section is a prism. Examples of prismatic structures in the wastewater industry include spoil piles, compost piles and tanks which have a triangular cross section in their floors for the purposes of collecting sludge or grit.

The equation for the volume of a prism is one half its base times its height times its length

The formula is

$$\text{Volume of a prism} = \frac{\text{base} \times \text{height}}{2} \times \text{length}$$

It is written:

$$V = \frac{B \times H}{2} \times L$$

Calculate the volume of a compost pile 3 metres high by 6 metres wide by 30 metres long.

Known: Base = 6 metres, Height = 3metres, Length = 30 metres

Insert known values and solve:

$$V = \frac{B \times H}{2} \times L = \frac{6 \text{ m} \times 3 \text{ m}}{2} \times 30 \text{ m} = 270 \text{ m}^3$$

Volume of a Cylinder

Calculation of the volume of a cylinder will probably be the most frequently used volume calculation after the calculation for the volume of a rectangular basin. Cylinders are found as circular clarifiers, reservoirs and water and sewer pipelines.

The volume of a cylinder is equal to the area of its circular base (the radius of the cylinder squared, multiplied by the constant π), multiplied by the height

The formula is

$$\text{Volume} = \pi \times (\text{radius})^2 \times \text{height}$$

It is written:

$$V = \pi r^2 h \text{ or } V = \pi r^2 H$$

What is the volume of a secondary clarifier that is 46 metres in diameter and 4.5 metres deep?

Known: Diameter = 46 metres therefore radius = 46÷2 = 23 metres, depth = 4.5 metres

Insert known values and solve

$$V = \pi r^2 h = 3.14 \times (23 \text{ m})^2 \times 4.5 \text{ m} = 7{,}474.7 \text{ m}^3$$

Volume of a Cone

Calculation of the volume of a cone is used less frequently but it may be required when calculating the volume of a storage hopper or the conical floor section of a digestor, clarifier or trickling filter.

The volume of a cone is equal to the one third (⅓) the area of its circular base (the radius of the cylinder squared, multiplied by the constant π), multiplied by the height

The formula is:

$$\text{Volume} = \frac{\pi \times (\text{radius})^2 \times \text{height}}{3} \quad or \;\; V = \frac{\pi r^2 h}{3}$$

Calculate the volume of conical hopper 2 metres deep and 1.5 metres in diameter.

Known: diameter = 1.5 metres, therefore radius = 1.5÷2=0.75 m, depth = 2 metres

Insert known values and solve:

$$\mathrm{V} = \frac{\pi \mathrm{r}^2\mathrm{h}}{3} = \frac{3.14 \times (0.75\ \mathrm{m})^2 \times 2\ \mathrm{m}}{3} = 1.18\ \mathrm{m}^3$$

Volume of a lagoon (a frustrum)

The correct name for a truncated pyramid is a frustrum. The EOCP handout provides a formula for calculating the volume of a lagoon which is a type of inverted truncated pyramid.

The volume of a frustrum is equal to one half (1/2) the average length times the average width times the depth.

The formula is:

$$\text{Volume} = \text{average length } \times \text{average width } \times \text{depth}$$

It is written:

$$\mathrm{V} = \frac{\mathrm{L_{top}} + \mathrm{L_{bottom}}}{2} \times \frac{\mathrm{W_{top}} + \mathrm{W_{bottom}}}{2} \times \text{depth}$$

Where L = length and W = Width

A lagoon measures 100 metres wide by 300 metres long on the surface, its bottom dimensions are 80 metres wide by 280 metres long. It is 2.5 metres deep. What is its volume?

Known: L_t = 300 m, L_b = 280 m W_t =100 m, W_b = 80m, Depth=2.5 m

Insert known values and solve:

$$\mathrm{V} = \frac{300\ \mathrm{m} + 280\ \mathrm{m}}{2} \times \frac{100\ \mathrm{m} + 80\ \mathrm{m}}{2} \times 2.5\ \mathrm{m} = 65{,}250\ \mathrm{m}^3$$